Assignment 7

1. The amount of kerosene, in thousands of liters, in a tank at the beginning of any day is a random amount \( Y \) from which a random amount \( X \) is sold during that day. Suppose that the tank is not resupplied during the day so that \( x \leq y \), and assume that the joint density function of these variables is

\[
f(x, y) = \begin{cases} 
2, & 0 < x < y, 0 < y < 1 \\
0, & \text{elsewhere}
\end{cases}
\]

a) Evaluate the marginal distribution of \( X \).

b) Evaluate the marginal distribution of \( Y \).

c) Determine if \( X \) and \( Y \) are independent.

d) Find \( P(\frac{1}{4} < x < \frac{1}{2} \mid y = \frac{3}{4}) \)

2. Let \( X \) denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let \( Y \) denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

<table>
<thead>
<tr>
<th>( f(x, y) )</th>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>0.05</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.05</td>
<td>0.1</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

a) Evaluate the marginal distribution of \( X \).

b) Evaluate the marginal distribution of \( Y \).

c) Determine if \( X \) and \( Y \) are independent.

d) Find \( P(y = 3 \mid x = 2) \)

e) Find \( \text{Var}(2x + 3y) \).

3. A coin is tossed twice. Let \( Z \) denote the number of heads on the first toss and \( W \) the total number of heads on the 2 tosses. If the coin is unbalanced and a head has a 40% chance of occurring, find

a) the joint probability distribution of \( W \) and \( Z \);

b) the marginal distribution of \( W \);
c) the marginal distribution of Z;
d) the probability that at least 1 head occurs.

4. The joint density function of the random variables X and Y is

\[ f(x,y) = \begin{cases} 
6x, & 0 < x < 1, 0 < y < 1 - x \\
0, & \text{elsewhere}
\end{cases} \]

a) Show that X and Y are not independent.
b) Find \( P(x > 0.3 \mid 0.1 < y < 0.5) \)
c) Find \( E[2x+y] \).

Due Date: 29.12.2011