AR spectral analysis of EEG signals by using maximum likelihood estimation

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Abstract

In this study, EEG signals were analyzed using autoregressive (AR) method. Parameters in AR method were realized by using maximum likelihood estimation (MLE). Results were compared with fast Fourier transform (FFT) method. It is observed that AR method gives better results in the analysis of EEG signals. On the other hand, the results have also showed that AR method can also be used for some other researches and diagnosis of diseases. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Power spectrum estimation methods have been used for a long time in the analysis of biological signals. From a historical perspective, it starts with the paper by Robinson and the book by Marple [1]. The classical power spectral estimation methods are mainly based on the fast Fourier transform (FFT), originally introduced by Schuster (1898). The modern model-based or parametric methods were originated by Yule (1927). These two methods were subsequently developed and applied by Walker (1931), Barlett (1948), Parzen (1957), Blackman and Tukey (1958), Burg (1967), and others [1].

EEG signals involve a great deal of information about the function of the brain. Since the classification and evaluation of these signals are limited, there is no definite criterion evaluated by the experts for the visual analysis of EEG signals. For example, in the case of dominant alpha activity,
delta and teta activities have not been noticed. Since the routine clinical diagnosis needs to analyze EEG signals, some automation and computer techniques have been used for this purpose. To do that, EEG signals have been analyzed by using different techniques [2,3].

Due to the random fluctuations of EEG signals, the statistical average characteristics of random signals should be obtained. In particular, the autocorrelation function of a random process is the appropriate statistical average that will be used for characterizing random signals in the time domain. The Fourier transform of the autocorrelation function, which yields the power density spectrum, provides the transformation from the time domain to the frequency domain and estimation of signal variance as a function of frequency.

Non-parametric power spectrum estimation methods can be calculated using FFT and understood easily by comparing with the parametric methods. Since FFT method needs long duration data records for suitable frequency resolution, it is applied to windowed data sets which assume all data zero except the data in the window. On the other hand, some spectral losses occur because of windowing process and these losses mask weak signals in the data [4]. Parametric power spectrum estimation methods reduce the spectral losses and give better frequency resolution. Also autoregressive (AR) method has an advantage over FFT in the case of shorter duration data records [5].

In order to model a signal, the properties of this signal must be taken into account. For example, AR model is suitable for the signals which have sudden peaks in their frequency spectrums. Inversely, moving average (MA) model is suitable for signals which have no sharp peaks in their frequency spectrums. On the other hand, the autoregressive-moving average (ARMA) model is suitable for signals which have both type of peaks in their frequency spectrums. Since EEG signals contain peaks at some frequencies, AR model can be used by employing Burg or Levinson–Durbin algorithm [6,7].

In this study, AR spectral analysis of EEG signals using maximum likelihood estimation (MLE) is studied. Data recorded from healthy and epileptic patients are processed using FFT and AR method with MLE in order to obtain the best method in the process.

2. Materials and method

2.1. Spectral analysis of EEG signals

EEG signals are analyzed by using spectral analysis methods to diagnose some cerebral diseases. The power spectral density of the signal $P(f)$ is found by applying conventional and modern spectral analysis methods such as FFT and AR. The data acquisition system for the processing of EEG signal is shown in Fig. 1.

EEG signals used in this study are taken from Medical Faculty Hospital of Dicle University. These signals belong to several healthy and unhealthy (epileptic patients) persons. The signals are collected by a data acquisition system which contains data acquisition card (PCI MIO-16-E+ type), signal processors and a personnel computer. Data can be taken into computer memory quickly by using this card which is connected to PCI data bus of the computer [8]. For this system C based graphical programming language (LabVIEW) is used. The system provides real time data processing.

Discrete Fourier transform (DFT) of a discrete signal $x(n)$ is defined as follows:

$$X_k = \sum_{n=0}^{N-1} x(n) \exp \left( -\frac{j2k}{N} \right),$$  

(1)
where $X_k$ is expressed as the discrete Fourier coefficient, $N$ is the frame size and $x(n)$ is the input signal on the time domain. To obtain the frequency spectrum of this signal, logarithmic values of the squares of absolute values of $X_k$ are found as follows:

$$P(k) = 10 \log |X|^2.$$  \hfill (2)

In the AR modeling method, the amplitude of a signal at a given period is obtained by summing up the different amplitudes of previous samples, and adding the estimation error. The AR coefficients, which identify the amplitude rates, can be calculated by using the Burg or Levinson–Durbin algorithms. In the Levinson–Durbin algorithm, AR parameters are found out by solving the Yule–Walker equations. However, in the Burg algorithm, AR coefficients are found out with advanced reversed errors, which depend on samples taken from the signal. The order of the model, namely the filter, depends on the number of AR coefficients. In the AR method, the model order is identified according to different criteria. In this study, Akaike information criteria (AIC) are taken as the base. After inspecting Refs. [7,9], the model order $p = 10$ was taken because the determined model order was lower. In the AR model, MLE is used for the solution of the Yule–Walker equations to get AR model parameters [9].

To obtain stable and high performance AR model, some factors must be taken into consideration such as: selection of the right algorithm according to the model, selection of the model order, the length of the signal which will be modeled, and the level of stationary of the data. The order of the model is very important. If the selected order is low, there will be no definite peaks in the spectrum. So, the frequency details of the signal cannot be identified. If the selected order is very high, faulty peaks may be seen in the frequency spectrum which is not related to the original signal.

2.2. Autoregressive parameter estimation and MLE

In the AR model, to find out model parameters of Levinson–Durbin algorithm which makes use of the solution of the Yule–Walker equations is used. Autocorrelation estimation is used for the solution of these equations. After those autocorrelation, AR model parameters are estimated. To do that, biased form of the autocorrelation estimation is used which is given as

$$r_{xx} = \frac{1}{N} \sum_{n=0}^{N-m-1} x^*(n)x(n+m), \quad m \geq 0.$$  \hfill (3)
The purpose now is to estimate the AR model parameters by using MLE in the solution of
the Yule–Walker equations from a record of EEG data. If the maximum likelihood estimate of
a parameter exists under regular condition, it is consistent, asymptotically unbiased, efficient, and
normally distributed. Unfortunately, the maximum likelihood (ML) estimator is often too cumbersome
to obtain [10]. As this is the case for the EEG model, it is proposed to estimate the model parameters
by maximizing an approximation of the log-likelihood function, known as Whittle’s approximation,
the derived estimator is expected to retain the properties associated with the ML estimator in an
asymptotic sense, but with much less complexity [4]. In fact, Whittle’s estimate asymptotically retains
the properties of the ML estimate for Gaussian random processes, but this is not generally true for
the non-Gaussian case [2].

In many cases, it is difficult to evaluate the MLE of the parameter its power spectrum density
function (PSDF) is Gaussian due to the need to invert a large dimension covariance matrix. For
example, if \( x \sim N(0, c(\theta)) \), the MLE of \( \theta \) is obtained by maximizing

\[
P(x; \theta) = \frac{1}{(2\pi)^{n/2} \det(c(\theta))} e^{(-1/2)x'c^{-1}(\theta)x}.
\]

(4)

If the covariance matrix cannot be inverted in the closed form, then a search technique will require
inversion of the \( N \times N \) matrix for each value of \( \theta \) to be searched. An alternative approximate method
can be applied when \( x \) is the data from a zero mean random process, so that covariance matrix is
Toeplitz. In such a case, the asymptotic log-likelihood function is given by

\[
\ln P(x; \theta) = \frac{N}{2} \ln 2\pi - \frac{N}{2} \int_{-1/2}^{1/2} \left[ \ln P_{xx}(f) + \frac{I(f)}{P_{xx}(f)} \right] df,
\]

(5)

where

\[
I(f) = \frac{1}{N} \left| \sum_{n=0}^{N-1} x(n)e^{-j2\pi fn} \right|^2
\]

is the periodogram of the data and \( P_{xx}(f) \) is the power spectral density (PSD). The dependence of
the log-likelihood function on \( \theta \) is through the PSD. Differentiation of (5) produces the necessary
conditions for MLE

\[
\frac{\partial \ln P(x; \theta)}{\partial \theta_i} = - \frac{N}{2} \int_{-1/2}^{1/2} \left[ \frac{1}{P_{xx}(f)} - \frac{I(f)}{P_{xx}^2(f)} \right] \frac{\partial P_{xx}(f)}{\partial \theta_i} df
\]

(6)

or

\[
\int_{-1/2}^{1/2} \left[ \frac{1}{P_{xx}(f)} - \frac{I(f)}{P_{xx}^2(f)} \right] \frac{\partial P_{xx}(f)}{\partial \theta_i} df = 0.
\]

(7)

The second derivative allows the Newton–Raphson or scoring method to be implemented using the
asymptotic likelihood function. This leads to simpler iterative procedures and is commonly used in
practice.

In this study, to find MLE, the asymptotic form of the log-likelihood given by (5) is used. Since the
PSD is

\[
P_{xx}(f) = \frac{\delta_u^2}{|A(f)|^2}
\]

(8)
after some calculations and derivations, the estimated autocorrelation function is \[3,4,10\]

\[
\hat{R}_{xx}(k) = \begin{cases} 
\frac{1}{N} \sum_{n=0}^{N-1-|k|} x(n)x(n + |k|), & |k| \leq N - 1, \\
0, & |k| \geq N
\end{cases}
\] (9)

and the set of equations to be solved for the approximate MLE of the AR filter parameters becomes

\[
\sum_{l=1}^{p} \hat{a}(l)\hat{R}_{xx}(k - l) = -\hat{R}_{xx}(k), \quad k = 1, 2, \ldots, p
\] (10)

or in matrix form

\[
\begin{pmatrix} 
\hat{R}_{xx}(0) & \hat{R}_{xx}(1) & \cdots & \hat{R}_{xx}(p - 1) \\
\hat{R}_{xx}(1) & \cdots & \cdots & \hat{R}_{xx}(1) \\
\vdots & \cdots & \cdots & \vdots \\
\hat{R}_{xx}(p - 1) & \hat{R}_{xx}(p - 2) & \cdots & \hat{R}_{xx}(0)
\end{pmatrix} 
\begin{pmatrix} 
\hat{a}(1) \\
\hat{a}(2) \\
\vdots \\
\hat{a}(p)
\end{pmatrix} = 
\begin{pmatrix} 
\hat{R}_{xx}(1) \\
\hat{R}_{xx}(2) \\
\vdots \\
\hat{R}_{xx}(p)
\end{pmatrix}.
\] (11)

These are the so-called estimated Yule–Walker equations and this is the autocorrelation method of linear prediction. Note that the special form of the matrix and the right-hand vector, which thereby allow a recursive solution known as the Levinson recursion [1]. To complete the discussion explicit form for the MLE of \(\delta_u^2\) should be determined. From (8)

\[
\delta_u^2 = \sum_{k=0}^{p} \hat{a}[k]R_{xx}[\cdot - k] = \sum_{k=0}^{p} \hat{a}[k]R_{xx}[k].
\] (12)

These parameters are used to compute the AR spectral power density as

\[
P_{xx}^{\text{YW}}(f) = \frac{\delta_w^2}{1 + \sum_{k=1}^{p} \hat{a}(k)e^{-j2\pi fk}}.
\] (13)

3. Results and discussions

In this study, analysis of three real and one simulated EEG signals is presented. For this aim, a software is developed by using laboratory virtual instrument engineering workbench (Lab View) Programming Language. These signals are analyzed by using FFT and AR method which have the MLE optimization. Results of the spectrums of these methods are represented below and compared with each other.

In Fig. 2 an epileptic EEG signal and FFT of this signal are given. If the frequency spectrum of FFT is examined, it is seen that there are peaks at 1 and 3 Hz. AR spectrum of the same signal is presented in Fig. 3. There are peaks at 3 Hz with higher amplitude, 6, 9.5 and 13.5 Hz. When we compare these two spectrums it is seen AR spectrum has got sharper peaks and less misleading peaks than FFT. Due to this better frequency solution, explanation and determination of the activities in the signal is easier by using AR method. Since the signal is taken from an epileptic patient, the results fit with the typical characteristics of epilepsy that is delta activity (low frequency range) [2].

Fig. 4 shows an EEG signal taken from a child and FFT of this signal. Spectrum of the AR method is given in Fig. 5. If these two spectrums are examined although FFT spectrum has got
Fig. 2. Epileptic EEG signal and its FFT.

Fig. 3. AR spectrum of epileptic EEG signal.

wide and misleading peaks AR spectrum has got sharp and clear peaks. If these spectrums are examined, the delta activity (1–3 Hz), the alpha activity (6–9 Hz), and the beta activity (11–13 Hz) can easily be seen. These results are true because it is a normal EEG signal. Higher variations and misleading peaks in FFT spectrum avoid the dominant alpha and delta activities.

Fig. 6 shows an EEG signal of a healthy person and its FFT spectrum. FFT spectrum has peaks at 3.5, 8 and 10 Hz. On the other hand, we can see nearly the same peaks in Fig. 7 but more clearly than FFT spectrum.

Fig. 8 shows a simulated EEG signal and its FFT spectrum. The signal contains four sinusoidal signals (at frequencies 2, 6, 9 and 15 Hz) and white noise. AR spectrum which uses the MLE is shown in Fig. 9. When these spectrums are compared with each other, it is seen that in the AR spectrum frequency content of the signal can be seen more clearly than FFT spectrum.
Fig. 4. Normal EEG signal taken from a three-year old child and its FFT.

Fig. 5. AR spectrum of EEG signal.

Fig. 6. EEG signal taken from a 29-year old healthy person and its FFT.
Fig. 7. AR spectrum of EEG signal.

Fig. 8. Simulated EEG signal and its FFT.

Fig. 9. AR spectrum of EEG signal.
4. Conclusions

In this study, the FFT and AR spectrums which have MLE optimization of real and simulated EEG signals are plotted and compared. To get AR method model parameters, MLE which has wide applications in statistics is used. After calculations and observations, it is seen that AR method spectrums have sharp and clear peaks. That is, AR spectrums show the frequency content of the signals more clearly than FFT spectrums and these results can be used for diagnosis of pathological events.

References


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