Comparison of STFT and wavelet transform methods in determining epileptic seizure activity in EEG signals for real-time application

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Abstract

Electroencephalography (EEG) is widely used in clinical settings to investigate neuropathology. Since EEG signals contain a wealth of information about brain functions, there are many approaches to analyzing EEG signals with spectral techniques. In this study, the short-time Fourier transform (STFT) and wavelet transform (WT) were applied to EEG signals obtained from a normal child and from a child having an epileptic seizure. For this purpose, we developed a program using Labview software. Labview is an application development environment that uses a graphical language G, usable with an online applicable National Instruments data acquisition card. In order to obtain clinically interpretable results, frequency band activities of δ, θ, α and β signals were mapped onto frequency–time axes using the STFT, and 3D WT representations were obtained using the continuous wavelet transform (CWT). Both results were compared, and it was determined that the STFT was more applicable for real-time processing of EEG signals, due to its short process time. However, the CWT still had good resolution and performance high enough for use in clinical and research settings.

Keywords: EEG; Wavelet transform; STFT; Epileptic seizure

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1. Introduction

Spectral analysis and signal decomposition continue to find wide use in a multitude of engineering disciplines. The basic idea in signal decomposition is to separate the spectrum into its constituent subspectral components and then process them individually, based on the application [1]. Electroencephalogram (EEG) is widely used clinically to investigate brain diseases. Since EEG signals contain a wealth of information about brain functions, there have been many attempts to apply spectral analysis techniques to EEG signals classification in brain abnormalities. To use this information in medical researches and illness diagnosis, spectral analysis of signals must be performed with spectral analysis methods and it is necessary to make automation [2]. The system shown in Fig. 1 was realized for this purpose.

Epileptic seizure may start at any period in life, under certain conditions it may also start nearly almost after birth. The first epileptic seizure is seen before 20 years, especially in the first 3 years and before adolescence time in three-fourth of all epilepsy patients [3]. Nowadays, EEG is accepted as the most economic and harmless technology in diagnosing this widespread illness. In most of the events it is used in determining that point in brain which causes the epilepsy, during the diagnosis if the abnormal excitation of nerve cells is decreased or not, and how fast it is decreasing.

EEG signals are not deterministic and they have no special formation like electrocardiogram (ECG) signals. Because of this, in the analysis of EEG signals, statistical and parametric analysis methods are used (such as time–frequency analysis, self relation, crosswise relation, wavelet transform). These methods provide imaging of frequency band at the moment of epileptic seizure. They also provide the determination of the time of frequency rhythm analysis of periodic EEG signals. Since their statistical properties are dependent on time and space, EEG signals are treated as complex signals. But these signals may be decomposed into typical sample periods analytically. Furthermore, if the temporal characteristics of EEG signals are taken into consideration, it will be seen that they are not stable as it is clearly seen from Fig. 11(a) [4–7].

Spectral analysis of the EEG signals is performed using the short-time Fourier transform (STFT), in which the signal is divided into small sequential or overlapping data frames and fast Fourier transform (FFT) applied to each one. The output of successive STFTs can provide a time–frequency representation of the signal. To accomplish this, the signal is truncated into short data frames by

\begin{center}
\includegraphics[width=\textwidth]{Fig_1.png}
\end{center}

Fig. 1. EEG signal acquisition and processing system.
multiplying it by a window so that the modified signal is zero outside the data frame. In order to analyze the whole signal, the window is translated in time and then reapplied to the signal.

The wavelet transform (WT) can be thought of as an extension of the classic Fourier transform, except that, instead of working on a single scale (time or frequency), it works on a multi-scale basis. This multi-scale feature of the WT allows the decomposition of a signal into a number of scales, each scale representing a particular coarseness of the signal under study. The WT provides very general techniques which can be applied to many tasks in signal processing. One very important application is the ability to compute and manipulate data in compressed parameters which are often called features. Thus, the EEG signal, consisting of many data points, can be compressed into a few parameters. These parameters characterize the behavior of the EEG signal. This feature of using a smaller number of parameters to represent the EEG signal is particularly important for diagnostic purposes.

In this study, the STFT and WT were applied to EEG signals obtained from 3- and 1.5-year-old children having epileptic seizures and a simulated EEG signal having 2.5, 6.5, 12.5 and 25 Hz frequency content as shown in Figs. 8(a)–11(a). To visualize the δ, θ, α and β frequency band activities depending on time and equal width surface of signals which is handled, signals are processed with time–frequency-based STFT and continuous wavelet transform (CWT) method. The applied methods were compared in terms of their frequency resolution and the effects in determining epileptic seizure activity in EEG signals.

2. Materials and method

2.1. Analysis of EEG signals

In order to detect frequency composition of the EEG signals and identify abnormalities, spectral analysis of the signals is performed. In this way, formation times of δ, θ, α and β frequency band activities in EEG signals are determined and the low-frequency content of δ band which is the most important part of signal according to epileptic seizure is visualized. To be able to achieve this aim, EEG signals are analyzed by the STFT and CWT methods in this study. For the application of these analysis methods, EEG signals in time domain are sampled at an appropriate frequency. Sampled signals are grouped as frames that contain evident sample numbers. The signals were processed and reconstructed by a system shown in Fig. 1 which was realized by our group in the previous study. For this purpose we developed a program using Labview software which is an application development program that uses a graphical programming language, G, to create programs as a block diagram form. Also it is usable with online applicable NI (PCI-MIO-16-E4) data acquisition card.

2.2. Short-time Fourier transform

Fourier analysis decomposes a signal into its frequency components and determines their relative strengths. We define the Fourier transform as

\[
F(w) = \int_{-\infty}^{\infty} f(t)e^{-jwt} dt \leftrightarrow f(t)
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w)e^{jwt} dw.
\]
This transform is applied to stationary signals, that is, signals whose properties do not evolve in time. When the signal is non-stationary we can introduce a local frequency parameter so that local Fourier transform looks at the signal through a window over which the signal is approximately stationary. Therefore, we applied the STFT to the EEG signals under study. The STFT positions a window function $\psi(t)$ at $\tau$ on the time axis, and calculates the Fourier transform of the windowed signal as

$$F(w, \tau) = \int_{-\infty}^{\infty} f(t)\psi^*(t - \tau)e^{-jwt} dt.$$  

(2)

When the window $\psi(t)$ is a Gaussian function, the STFT is called a Gabor transform. The basis functions of this transform are generated by modulation and transformation of the window function $\psi(t)$, where $w$ and $\tau$ are modulation and translation parameters, respectively. The fixed time window $\psi(t)$ is the limitation of STFT as it causes a fixed time–frequency resolution. This is explained by the uncertainty principle (Heisenberg inequality—meaning one can only trade time resolution for frequency resolution, or vice versa) for the transform pair

$$\psi(t) \leftrightarrow \Psi(w):$$

$$\Delta t \Delta w \geq \frac{1}{2},$$

(3)

where $\Delta w$ and $\Delta t$ are the bandwidth and time spread (i.e. two pulses in time can be discriminated only if they are more than $\Delta t$ apart) of $\psi(t)$, respectively, and

$$\Delta t^2 = \frac{\int t^2|\psi(t)|^2 dt}{\int |\psi(t)|^2 dt},$$

$$\Delta w^2 = \frac{\int w^2|\Psi(w)|^2 dw}{\int |\Psi(w)|^2 dw}.$$  

(4)

When $t$ increases, the window function translates in time. On the other hand, the increase in $w$ causes a translation in frequency with a constant bandwidth [6–9].

Since all simulated and original EEG data had 200 samples, and, this was not enough to perform the STFT and admissible representation of output transform, a linear interpolation process was applied to the data. A convenient linear interpolated value was inserted between every neighboring data. By this process data samples increased to 800 and all the rectangular windows in use had 16 samples length. Figs. 8(b)–11(b) shows the STFT power spectrum outputs of the EEG signals.

2.3. Wavelet transform analysis

There are several types of wavelet transforms, and, depending on the application, one may be preferred to the others. For a continuous input signal, the time and scale parameters can be continuous, leading to the continuous wavelet transform (CWT).

They may as well be discrete leading to a Wavelet Series expansion. Finally, the wavelet transform can be defined for discrete-time signals, leading to a discrete wavelet transform (DWT).

In the discrete time case, two methods were developed independently namely pyramidal coding or multiresolution signal analysis and sub-band coding. Note that the scale parameter in discrete wavelet analysis is to be understood as follows: For large scales dilated wavelets take “global views” of a subsampled signal, while for small scales, contracted wavelets analyze small “details” in the signal.
As a high pass filtered version of $x(n)$ (using a filter with impulse response $h(n)$), followed by subsampling by two. The low pass, subsampled approximation is obtained.

In this study, we want to choose an arbitrary band weight and center frequency for the wavelets’ filter banks, and therefore we used CWT instead of DWT.

In particular, the WT is of interest for the analysis of non-stationary signals, because it provides an alternative to the classical STFT. The basic difference is in contrast to the STFT, which uses short windows at high frequencies and long windows at low frequencies. This is in the spirit of the so-called “constant-Q” or constant relative bandwidth frequency analysis. It is desirable to see the WT as a signal decomposition onto a set of basis functions. In fact, as mentioned previously, these basis functions are called wavelets. They are obtained from a single prototype wavelet by dilations and contractions (scaling) as well as shifts. The prototype wavelet can be thought of as a band-pass filter, and the constant-Q property of the other band-pass filters (wavelets) follows because they are scaled versions of the prototype. Therefore, in WT, the notion of scale is introduced as an alternative to frequency, leading to a so called time-scale representation. This means that a signal is mapped into a time-scale plane.

To overcome the resolution limitation of the STFT, one can imagine letting the resolution $\Delta t$ and vary $\Delta f$ in the time–frequency plane in order to obtain a multiresolution analysis. Intuitively, when the analysis is viewed as a filter bank, the time resolution must increase with the central frequency of the analysis filters. We therefore impose that $\Delta f$ is proportional to $f$, or

$$\frac{\Delta f}{f} = c,$$

where $c$ is a constant. The analysis filter bank is then composed of band-pass filters with constant relative bandwidth. When the above equation is satisfied, we see that $\Delta f$ and therefore also $\Delta t$ changes with the center frequency of the analysis filter. Of course they will satisfy the Heisenberg equation, but now, the time resolution becomes arbitrarily good at high frequencies, while the frequency becomes arbitrarily good at low frequencies. For example, two very close short bursts can always be eventually separated in the analysis by going up to higher analysis frequencies in order to increase time resolution. This kind of analysis of course works best if the signal is composed of high-frequency components of short duration plus low-frequency components of long duration.

A generalization of the concept of changing resolution at different frequencies is obtained with so called “wavelet packets”, where arbitrary time–frequency resolutions are chosen depending on the signal [1,7].

To take the WT of a signal we have a mother wavelet

$$\psi_{a,b}(t) = \left( \frac{1}{\sqrt{a}} \right) \psi \left( \frac{t-b}{a} \right).$$

(7)

Then the WT of a function $f(t)$ is

$$W_{\psi}f(a,b) \equiv \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \psi^* \left( \frac{t-b}{a} \right) dt = \langle f, \psi_{a,b} \rangle.$$

(8)

As mentioned above, the WT is called continuous if the scaling and translation parameters $a$ and $b$, respectively, are continuous.

The CWT has two drawbacks: redundancy and impracticality. These problems are solved by making discretize the transform parameters $(a,b)$. 
In much of the wavelet transform literature, orthogonal, dyadic functions are chosen as the mother wavelet. This transform is often discretized in \( a \) and \( b \) on a dyadic (powers of two) grid, with time \( t \) remaining continuous. This discretization is performed by selecting \( a = a_0^j \) and \( b = k b_0 a_0^j \), where \( j, k \in \mathbb{Z} \), \( a_0 > 1 \) is a fixed dilation parameter, and \( b_0 \) is a fixed translation parameter.

Let the parameter sampling lattice be as previously stated: \( a = a_0^j \); \( b = k b_0 a_0^j \) so that

\[
\psi_{j,k}(t) = (a_0^{-j/2})\psi(a_0^{-j}t - k b_0),
\]

where \( j, k \in \mathbb{Z} \). If this set is complete in \( L^2(\mathbb{R}) \) for some choice of \( \psi(t), a, b \), then the basis functions \( \{\psi_{j,k}\} \) are called affine wavelets. Hence we can express any \( f(t) \in L^2(\mathbb{R}) \) as the superposition

\[
f(t) = \sum_j \sum_k d_{j,k} \psi_{j,k}(t),
\]

where the wavelet coefficient

\[
d_{j,k} = \langle f(t), \psi_{j,k}(t) \rangle = \frac{1}{a_0^{j/2}} \int f(t) \psi(a_0^{-j}t - k b_0) \, dt.
\]

The following is an analogy: assume that the wavelet analysis is like a microscope. First, one chooses the magnification, that is \( a_0^{-j} \). Then one moves to the chosen location. Now, if one looks at very small details, then the chosen magnification is large and corresponds to \( j \) negative and large. Then \( a_0^j b_0 \) corresponds to small steps, which are used to catch small details [1,4,7].

As a result, as we expressed in different styles before, because of it has pre-focus (focus) property on short time high-frequency events realized in on signals in medical signal process, particularly in researching epileptic seizure of EEG signals, WT is very useful in the expression of discontinuities caused by recording apparatus [2,6,9–17].

### 2.3.1. Continuous wavelet analysis

To achieve CWT the Mexican Hat function \( \psi(t) \) is used as a mother wavelet. This is a smooth, symmetric and having a compact support properties wave. A dilated and translated version of this mother wavelet gives us the constant \( Q \) band–pass filters also to obtain a multiresolution analysis.

\[
\psi(t) = (1 - t^2) e^{-t^2/2}.
\]

To avoid redundancy and impracticality properties of this method the transform parameters \((a, b)\) are discretized by using a simplified form of Eq. (9) where \( a_0 = 2 \) and \( b_0 = 1 \). At this time it is important to choose the parameters of transform especially \( j \), is dilation or scale parameter. Now, reconstruction of the original data is dependent on transform accuracy. Because the data and wavelet convolution give detailed coefficients of the transform, wavelets have to fit the data with dilation or scale parameter \( j \). A special discretized wavelet form used in this study is shown in Eq. (13). The lowest frequency resolution or the biggest scale parameter was set to 8 that fitted data all over 200 samples and redundancy was decreased by using Eq. (13). Also, this way \( M \) scale resolution was independent and changeable freely. Bigger choice of \( M \) only affected computation time of process.

\[
\psi_{j,k}(t) = (2^{-(8^j/M)^2})\psi(2^{-(8^j/M)}t - k) \quad j = 0, \ldots, M.
\]

This form of wavelet depends on used data or data specific, and, provides an approximation to perfect reconstruction of the original data as it is seen in Fig. 6. Fig. 2 shows center frequencies of
constant $Q$ wavelets. All the produced wavelets have constant, $Q = 3.1$. One can make some basic evaluations and simplified interpretation on transform outputs by examining Fig. 2 and $Q$ coefficient.

Figs. 2–7 show an instantaneous view of the output of developed software for scale parameter $j = 6$. An instantaneous convolution process is shown in Fig. 3.

Fig. 4 represents power spectrum of the wavelet for $j = 6$ and gives a filter having 2.5 Hz center frequency and constant $Q = 3.1$. This has basically very good band-pass filter characteristics. By using this filter the original signal was decomposed as shown in Fig. 5 and reconstructed by using decomposed elements as shown in Fig. 6.

In order to obtain clinically interpretable results for medicine, $\delta$, $\theta$, $\alpha$ and $\beta$ frequency band activities of signals are investigated.

Simulated data were used which had equal amplitude and three components in every $\delta$, $\theta$, $\alpha$ and $\beta$ frequency bands. $M = 32$ and first scale $j = 0.25$ to fetch or decompose high-frequency component of signal. Fig. 7 shows center frequency of wavelets filters and related EEG $\delta$, $\theta$, $\alpha$ and $\beta$ frequency
bands with scale or dilation of mother wavelet. δ band activity between 0.5 and 4 Hz can be resolved from CWT for scales between 10 and 32. θ band activity is between 4 and 8 Hz can be resolved from CWT for scales between 6 and 10. α band activity is between 8 and 13 Hz can be resolved from CWT for scales between 4 and 6. β band activity is between 13 and 22 Hz can be resolved from CWT for scales between 3 and 4.

3. Experimental result and discussion

The signals used in this study were real EEG signals and recorded in Neurology Laboratory D.U. Recordings were made along with a data-collecting unit developed in the previous study by our
During the recordings, silver surface electrodes were used and C3-A2 standard settlement was applied to the subject. All signals were digitized and transferred to the PC using 12-bit NI (PCI-MIO-16-E4) data acquisition card. The sampling frequency of the signal recorded during 6 s was 33 Hz. Because the required band activities were less than 4 Hz for the epilepsy activity, sampling frequency was chosen to be low. The STFT power spectrum and WT of the real EEG signals were obtained with Labview by creating programs in block diagram form.

The frame length was chosen as 16 for the STFT. Since all simulated and original EEG data had 200 samples, and this was not enough to perform the STFT and admissible representation of output transform, a linear interpolation process was applied to the data. Also the STFT power spectrum outputs have normalized with 256 to provide an admissible color scale representation. A convenient linear interpolated value was inserted between every neighboring data. By this process
samples increased to 800 and all the rectangular windows in use had 16 samples length. Figs. 8–11 show the EEG signals and the outputs of the STFT power spectrum and the CWT, respectively. The STFT and CWT spectra were placed on the same figure, so the comparison of the results was simplified.

Fig. 8(b) shows the band activities of the simulated EEG signal in 3D frequency–time domain obtained by the STFT for 6 s time interval and 800 samples with a window length of 16. Time–frequency distribution of the STFT output is ranged from 0 to 14 Hz. As it is seen from this figure, higher magnitude distribution was observed with low frequencies and lower magnitude distribution was observed with high frequencies. Transition from high- to low-frequency region occurs smoothly. This case approximately observed in the STFT of EEG signal obtained from a 3-year-old healthy child (Fig. 11(b)). Otherwise in case of epileptic seizure, transition from high- to low-frequency region occurs sharply and low-frequency distribution is more dominant as seen in Figs. 9(b) and 10(b).

To determine the forming time of $\delta$, $\theta$, $\alpha$, and $\beta$ frequency band activities in EEG signals and to visualize the low-frequency content of $\delta$ band which is the most important part of signal according to “epilepsy” seen on children; signals were decomposed into approximation (high scale, low frequency) and detail (low scale, high frequency) coefficients mentioned often in WT analysis. The 3D image
Fig. 9. (a) EEG signal obtained from a 1.5-year-old child having epileptic seizure. (b) STFT of EEG signal. (c) CWT of EEG signal.

Time-scale presentation of the CWT output of the simulated EEG signal is shown in Fig. 8(c). 3D presentation of the CWT output of the EEG signal obtained from a 1.5-year-old child scaled from 0 to 32 as seen from Fig. 9(c). 3D presentation of the CWT of EEG signal obtained from a 3-year-old healthy child is shown in Fig. 11(c), where transition from high- to low-frequency region occurs smoothly and magnitude difference between them is not large. From Figs. 9(c) and 10(c) in the case of epileptic seizure higher magnitude distribution was observed with range of high scale-low frequencies and lower magnitude distribution was observed with range of low scale-high frequencies. Transition from high- to low-frequency region occurs sharply and low-frequency distribution is more dominant. Thus, comparison and interpretation of the EEG signals can be made easily.

4. Conclusion

In this study, the STFT and WT spectral analysis methods were compared in terms of their frequency resolution and the effects in determining epileptic seizure activity in EEG signals. In order
Fig. 10. (a) EEG signal obtained from a 3-year-old child having epileptic seizure. (b) STF of EEG signal. (c) CWT of EEG signal.

to obtain clinically interpretable results for medicine, frequency band activities of $\delta$, $\theta$, $\alpha$ and $\beta$ signals were plotted 3D frequency–time axis by using the STFT methods and 3D WT representation by using CWT which is a multiresolution analysis technique. To achieve this aim, first, real EEG signals were obtained from subjects by using a data acquisition and processing unit (PCI-MIO-16-E4) and a personal computer, and then, Labview software was used to evaluate these spectral analysis methods.

The WT is of interest for the analysis of non-stationary signals, because it provides an alternative to the classical STFT. When these methods were compared in term of process time, the STFT took the shortest time and it was applicable on line. On the other hand, the CWT took longer time but it produced more accurate result in recognizing of the EEG signals. Especially, detection of the epileptic seizure and classification of EEG signals with wavelet is useful because adjustment of wavelet analysis parameters (e.g., bandwidth and central frequency of filter banks via scaling) and the evaluation of the results obtained with new techniques such as neural networks give better results than STFT.

Based on these results, it can be mentioned that the WT had good resolution and high performance for visualisation of the epilepsy activity and it can be used in clinical and research areas.
Fig. 11. (a) EEG signal obtained from a 3-year-old healthy child. (b) STFT of the EEG signal. (c) CWT of the EEG signal.

References


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