On the Performance Comparison of Non-ideal Single-Carrier and Multi-Carrier Waveforms over Wide-band HF Channels

F. Genc*, M. A. Reşat*, A. Yavanoğlu*, and Ö. Ertuğ*

Gazi University
Telecommunications and Signal Processing Laboratory
Electrical and Electronics Engineering Department
Ankara, Turkey

Abstract—The purpose of this paper is to quantify the effects of Carrier Frequency Offset (CFO), Symbol Timing Offset (STO) and Phase Noise in coded Orthogonal Frequency Division Multiplexing (OFDM) and coded Single Carrier Frequency Domain Equalization (SC-FDE) systems and to compare their relative sensitivities in wide-band HF channel model. In mobile communication systems where multi-path propagation occurs channel estimation and equalization is additionally necessary. Furthermore a non-ideal local oscillator generally is misaligned with the operating frequency at the receiver and this causes carrier frequency offset (CFO). Therefore in coded SC-FDE and coded OFDM systems; a very efficient, low complex frequency domain channel estimation and equalization is implemented in this paper. Also Cyclic Prefix (CP) based synchronization synchronizes the clock and carrier frequency offset. Proposed models show that, one of the most powerful code, Turbo Code, enables to utilize the full frequency diversity available in the channel. The BER performance of the computer simulations specify that non-ideal turbo coded OFDM has better performance with greater diversity than non-ideal turbo coded SC-FDE system in wide-band HF channel.

I. INTRODUCTION

Due to the constrained frequency bandwidth, potential of power and spectral efficiency of mobile and base stations are reasonably expected as high as possible at the future communication systems. New air interfaces need to be improved to meet the new user necessities. To satisfy the requirements, Orthogonal Frequency Division Multiplexing (OFDM) is a popular modulation technique adopted to broadcast systems, such as Digital Video Broadcasting (DVB), Digital Audio Broadcasting (DAB), for wireless low-power systems Wireless Local Area Networks (WLAN) and Asymmetric Digital Subscriber Line (ADSL) for wired systems. In OFDM systems, instead of sending the information as a single stream an OFDM transmitter divides the information into many low-rate streams which are transmitted parallel [1]. One of the main disadvantages of the OFDM system is a large Peak-to-Average Power Ratio (PAPR) resulting from the addition of independently modulated sub-carriers. Even though Single Carrier Frequency Domain Equalization (SC-FDE) is an desirable alternative to the OFDM system. SC-FDE experience a lower PAPR than OFDM because no Inverse Fast Fourier Transform (IFFT) is performed at the transmitter to pre-code the signal. In order to mitigate the PAPR problem, Single Carrier (SC) transmission uses single carrier modulation instead of many sub-carriers [1-3]. In additionally, another difference is computational complexity required at the receiver and at the transmitter. Considering the OFDM system, one IFFT block is performed at the transmitter and also one FFT operator is performed at the receiver sides of the link. In case of SC-FDE system, no IFFT and FFT block is existed at the transmit side while two FFT/IFFT operators are performed at the receiver side of the link. Therefore SC-FDE receiver has a complex structure than OFDM receiver.

For the low-complexity approach to Inter Symbol Interference (ISI) mitigation both system benefit from low-complexity multi-path channel equalization and estimation in the frequency domain [4]. For this purpose to minimize the attenuations of the fading channel MMSE equalizer is used. At the same time MMSE estimation is used with the comb-type pilot tone arrangement to predict the multi-path channel propagation effects [9-11]. In order to keep track of time-varying channel characteristics, the comb-type pilot symbols are placed as frequently as the coherence time that is inverse form of the Doppler frequency shift in the channel. Additional way to eliminate ISI almost completely, is a guard interval which is called CP. CP of length $L$ symbols is formed by adding a repetition of the last $L$ transmitted symbols in a block to the beginning of the block before transmission. The guard time $L$ must be larger than the expected channel delay spread. Next at the receiver, the received CP is discarded before precessing the block.

Contemporaneously CP is used in CP-based channel synchronization to compensate the Inter Carrier Interference (ICI) is caused by Doppler effect. CP-based synchronization enables to CFO estimation without need of additional redundant pilots. In fact the key element is the CP already contains sufficient information to perform synchronization. Without a CFO, the sub-channels do not interfere with one another. The impact of a frequency offset is a loss of orthogonality between the each
tones. Hence, received signal is not a white process because of its probabilistic structure, it contains information about the time offset and carrier frequency offset [12]. Estimation of timing offset $\theta$ and frequency offset $\varepsilon$ are achieved by relation of each CPs of consecutive frames.

In this paper the performance of the SC-FDE and OFDM system in wide-band HF channel communication is evaluated. In practice a wide-band radio channel is time-variant, frequency-selective and noisy. The HF channel model is recommended by CCIR and ITU-R that is the most commonly used narrow-band Watterson HF channel model [5,6]. Main restriction of the Watterson model is that the model is designed and tested for narrow-band not more 12KHz bandwidth. In the design of high data speed wide-band HF communication systems, exact modeling and simulation of HF channel are needed. So Vogler-Hoffmeyer HF channel model is used in this paper [7,8].

The objective of this paper is to evaluate the affectability of SC-FDE system to CFO, STO and phase noise instability, and to compare it to the sensitivity of the OFDM system. The effect of the non-idealities on the SC-FDE system is not generally tended in the literature. This work note that for turbo-coded OFDM performs significantly better than turbo-coded SC-FDE in HF channel model with the large diversity. The basis of diversity is to take advantage of signals that are not correlated. The results confirm that turbo code satisfy to achieve full diversity. The minimum Hamming distance $d_{free}$ (free distance) of the turbo code depends on its coding rate and must be greater than the channel orders. For the 1/3 code rate, minimum distance of the turbo code is 5 which is greater than the channel order 3-taps, to achieve full diversity.

The simulation parameters are compliant to the wide-band HF channel model: 24KHz bandwidth, 16 QAM constellation, 256 sub-carriers, 210 occupied sub-carriers, 16 cyclic prefix length and pilot tone number is equal to 30. The code rate of the turbo code is 1/3, the interleaver is 512 bit block interleaver and 10-iteration log-MAP decoding is used.

The remainder of this paper is organized as follows: Section II gives an overview of wideband HF channel model. Section III overviews Orthogonal Frequency Domain and Single Carrier structure. In Section IV Channel Estimation and Synchronization methods are defined in detail. Simulation results are given in Section V, which shows research results of each OFDM and SC-FDE BER performance that is parametrized by channel delays, doppler shifts and phase noise. Finally, conclusions are drawn out from the achieved results.

II. WIDE-BAND HF CHANNEL MODEL

HF channel provides opportunity for long-distance communication via ionospheric reflections. So, HF channel characteristics are formed by the behavior of the ionosphere.

The wide-band HF channel is a FIR filter where each taps have time varying property and are in the complex domain. That model can be expressed by the following equation:

$$y_t = \sum_{i=0}^{L-1} h_i x_t + n_t \quad (1)$$

where $y_t$ is the complex output of the channel, $x_t$ is the complex input to the channel, $h_i$ is one of the $L$ taps of the time-varying transversal filter, $L$ is the length of the channel and $n_t$ is Additive White Gaussian Noise (AWGN). By the way a Complex-Valued FIR filter can be easily formed with convolution of the input signal with the channel impulse response. So the coefficients of the FIR filter are the samples of the channel impulse response of the HF channel which is given as:

$$h(t, \tau) = \sqrt{P(\tau)} D(t, \tau) \psi(t, \tau) \quad (2)$$

where $\sqrt{P(\tau)}$ is the delay power profile, and its square root $\sqrt{P(\tau)}$ describes the shape in delay dimension; $D(t, \tau)$ is the deterministic phase function describing the Doppler shift of each path, and $\psi(t, \tau)$ is stochastic modulation function which is the fading of the impulse response.

The doppler effect can be given as:

$$D(t, \tau) = e^{j2\pi f_D t} \quad (3)$$

where $f_D$ is the doppler shift. The stochastic modulation function $\psi(t, \tau)$ can be denoted as random variables which have an auto-correlation function with Gaussian shape.

The wide-band HF channel model with propagation paths structure is determined in Fig.1 and each of single propagation path model is shown in Fig.2 [8]. It is important to specify that the main difference between the narrow-band Watterson model and wide-band channel model is that time delay spread is neglected in the Watterson model and the time delay of each path has a single value. However the wide-band model has a delay power profile denoted by $P(\tau)$ and doppler effect is related with the time delay of the path.
where the constant \(1\) in order to eliminate inter symbol interference (ISI). Next guard time includes the cyclic extended part of OFDM symbol must be greater than the maximum channel delay and the conversion output of the IDFT can be expressed as:

\[
x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}
\]  

III. SYSTEM MODELS

A. OFDM System

In this section proposed OFDM system model is described and system structure is illustrated at Fig. 3. First each binary source datas are encoded by non-punchered, \(R = 1/3\) code-rate Turbo encoder and Log-Map algorithm is chosen for best performance and low-complexity for Turbo decoder. In this time N subcarrier \(X_k\) for \(k=0,1,....N-1\) are modulated by a signal alphabet \(A\) used for transmitting the information for 16-QAM \(A = \{\pm 1, \pm 3, \pm i, \pm 3i\}\) and \(j = \sqrt{-1}\). After baseband modulation, pilot tone symbol insertion is applied for the channel estimation where the pilot pattern is shown in Fig. 4.

Pilot arrangement in OFDM system issue is discussed more detaily under the Section IV. Here after the serial to parallel conversion output of the IDFT can be expressed as:

\[
x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}
\]  

where the constant \(\frac{1}{\sqrt{N}}\) normalizes the power \(N\) is the subcarrier number and \(X(k)\) is the modulated input symbols.

Next \(N_c\) is the cyclic prefix (CP) length is appended which must be greater than the maximum channel delay and the composite signals are transmitted to the HF channel. This guard time includes the cyclic extended part of OFDM symbol in order to eliminate inter symbol interference (ISI). Next the RRC Pulse Shaping Filtering is reconstructed the data symbols.

When FFT is taken at the receiver the received signal given by

\[
Y[k] = \sum_{n=0}^{N-1} y[n] e^{-j2\pi kn/N}
\]

\[
= \sum_{n=0}^{N-1} \left( \sum_{m=0}^{\infty} h[n] x[n-m] + z[n] \right) e^{-j2\pi kn/N}
\]

\[
= \sum_{n=0}^{N-1} \left( \sum_{m=0}^{N-1} h[m] \left( \frac{1}{N} \sum_{i=0}^{N-1} X(i) e^{j2\pi (i-n)/N} \right) \right) e^{-j2\pi kn/N} + Z[k]
\]

\[
Y[k] = H[k]X[k] + Z[k]
\]  

where \(X[k]\) denotes the \(k\)th subcarrier frequency components transmitted symbol, \(Y(k)\) is received symbol , \(H[k]\) is channel frequency response and \(Z[k]\) is noise in frequency domain, respectively.

At the receiver, after passing to discrete-time domain through A/D converter and pulse shaping filter, CP-based ML synchronization is applied to compensate the Carrier Frequency Offset (CFO) which is mentioned at section IV then guard time is removed:

\[
y[n] = \{ y_g(n) \quad M < n < N \}
\]

where \(N\) is the subcarrier number, \(M\) is CP length, \(y_g\) received signal that have guard interval insertion.

Then \(y_n\) is received to DFT block for the following operation:

\[
Y[k] = DFT\{y(n)\}, \quad k, n = 0, 1, ...N - 1
\]

\[
= \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j2\pi kn/N}
\]  

Next Least Square estimated \(\hat{H}_k^{LS}[k] = \frac{Y^p[k]}{X^p[k]}\) is obtained by extracting the pilot signals \(Y^p[k]\) . The interpolated \(\hat{H}[k]\) for all data subcarriers is obtained in MMSE channel estimation. Then, in the Frequency domain equalization (FDE) block the transmitted data is equalized by MMSE equalizer as below:

\[
\hat{X}_n = IDFT\{Y_n C_k\} = y_n \otimes c_n
\]  

where \(C_k\) represents the equalizer correction term, which is computed according to the FDE as follows:

- **MMSE Equalizer:**

\[
C_k = \frac{\hat{H}_k^{DFT}}{\left| \hat{H}_k^{DFT} \right|^2 + (E_b/N_0)^{-1}}
\]

where \((.)^*\) denotes conjugate. MMSE equalizer given in (9) takes into account the signal-to-noise ratio (SNR) making an optimum trade-off between channel inversion and noise enhancement [4]. Finally the binary information data is obtained back in 16-QAM modulation and Turbo decoding respectively.
B. SC-FDE System

As shown in Fig. 5, the main difference between OFDM and SC systems is the placement of the IDFT block. In SC systems, it is placed at the receiver side to transform the frequency domain equalized signals, thus compensating for channel distortion, bringing back to the time domain [3]. At both sides of the transmission all the other blocks are formed with the same manner as OFDM systems.

In the OFDM system, symbols are exposed to an additional transformation by using the IDFT, \( x(n) = IDFT\{X[k]\} \), but in the SC-FDE system no transformation is existed. Notice that the frame of SC-FDE is transmitted during the time instant after the Turbo encoder, 16-QAM modulation, Pilot insertion and CP insertion are applied respectively and the receiver maps received data into the frequency domain in order to equalize. When the channel delay spread is large, it is more efficient computationally to equalize in the frequency domain. In addition, SC-FDE has better behavior when used with non-linear power amplifiers.

IV. CHANNEL ESTIMATION & SYNCHRONIZATION

A. Channel Estimation

Figure 6 shows the pilot arrangement of the comb-type which is used for frequency domain interpolation to estimate the channel frequency response that is the Fourier transform of the channel impulse response [9, 10]. In Comb-type pilot arrangement every OFDM and SC symbol has pilot tones where are periodically located at the each subcarriers. Notice that \( S_f \) the periods of pilot tones in frequency domain which must be placed in the coherence bandwidth as the coherence bandwidth is determined by an inverse of the maximum delay spread \( \sigma_{\text{max}} \). The pilot symbol period is expressed as following inequality:

\[
S_f = \frac{1}{\sigma_{\text{max}}} \tag{10}
\]

Let consider the \( \hat{H}_{LS} = X^{-1}Y \triangleq \hat{H} \), using the weight matrix \( W \) channel estimate \( \hat{H} \triangleq W\hat{H} \) is defined and MSE of the channel estimate is calculated as below:

\[
J(\hat{H}) = E\left\{\| e \|^2 \right\} = E\left\{\| H - \hat{H} \|^2 \right\} \tag{11}
\]

Figure 7 shows the block diagram of DFT-based channel estimation, given the MMSE channel estimation. Notice that \( \sigma_{\text{max}} \) must be known in advance for eliminating the effect of noise outside the channel delay. Taking the IDFT of the MMSE channel estimate \( \hat{H} \) to get in the time domain, that the coefficients contain the noise are ignored with zero padding and then transform the remaining \( \sigma_{\text{max}} \) elements back to the frequency domain to achieve \( \hat{H}_{DFT} \). Finally \( \hat{H}_{DFT} \) is used in (9) at the Frequency Domain MMSE Channel Equalizer block.

Figure 8 shows the block diagram of DFT-based channel estimation, given the MMSE channel estimation. Notice that \( \sigma_{\text{max}} \) must be known in advance for eliminating the effect of noise outside the channel delay. Taking the IDFT of the MMSE channel estimate \( \hat{H} \) to get in the time domain, that the coefficients contain the noise are ignored with zero padding and then transform the remaining \( \sigma_{\text{max}} \) elements back to the frequency domain to achieve \( \hat{H}_{DFT} \). Finally \( \hat{H}_{DFT} \) is used in (9) at the Frequency Domain MMSE Channel Equalizer block.
B. Channel Synchronization

In general, there are two types of distortion related with the carrier signal. One is the Phase Noise due to the Voltage Control Oscillator (VCO) and the other is Carrier Frequency Offset (CFO) is caused by Doppler Frequency shift $f_d$. Let define the normalized CFO, $\varepsilon$, as a ratio of the CFO to subcarrier spacing $\Delta f$ is shown as:

$$\varepsilon = \frac{f_d}{\Delta f}$$  \hspace{1cm} (15)

where $f_d$ is the Doppler Frequency.

CP-based channel synchronization estimates the time and carrier-frequency offset. This algorithm uses the cyclic prefix of the OFDM and SC symbols, thus reducing the need of pilots. The received data in the time domain $e^{j2\pi f_k/N}$, where $\varepsilon$ denotes the difference in the transmitter and receiver oscillators as a fraction of the inter-carrier spacing , that is calculated in (15). Notice that all subcarriers are effected by the same shift $\varepsilon$ is shown as:

$$r(k) = s(k - \theta)e^{j2\pi f_k/N} + n(k)$$  \hspace{1cm} (16)

where $r(k)$ is the received data, $s(k - \theta)$ is the transmitted signal with unknown arrival time $\theta$ and $n(k)$ is the AWGN. Hence $r(k)$ contains information about the time offset $\theta$ and carrier frequency offset $\varepsilon$. From the observation shown in Figure.9 that the estimation of timing offset $\hat{\theta}$ and the estimation of frequency offset $\hat{\varepsilon}$ calculated as below:

$$\gamma(m) \triangleq \sum_{k=m}^{m+L-1} r(k)r^*(k + N)$$

$$\Phi(m) \triangleq \frac{1}{2} \sum_{k=m}^{m+L-1} |r(k)|^2 + |r(k + N)|^2$$

$$\hat{\theta}_{ML} = \arg \max_{\theta} \left\{ \frac{1}{SNR} \frac{\gamma(\theta)}{\Phi(\theta)} \right\}$$

$$\hat{\varepsilon}_{ML} = -\frac{1}{2} \ln \left( \frac{\gamma(\hat{\theta}_{ML})}{\Phi(\hat{\theta}_{ML})} \right)$$  \hspace{1cm} (17)

where $L$ is the CP length, $m$ is the index of samples, $\gamma(m)$ is the correlation coefficient and $\Phi(m)$ is an energy term [11,12]. Figure.9 shows the estimation of timing offsets and frequency offsets.

Finally these estimates are used in channel synchronization block to compensate the carrier frequency offset as:

$$\hat{s}(k) = r(k) \cdot e^{-j2\pi \hat{\varepsilon}k/N}$$  \hspace{1cm} (18)

where $\hat{s}(k)$ is the synchronized signal.

V. Numerical Results

In this section BER performance of the proposed systems for CFO, STO and phase noise are shown. In all of the simulations, modulation scheme 16-QAM is used. CP length is 16 symbols and normalized frequency offset of each system is a constant value between 0.1 and 0.5. For the channel model multipath fading channel is used which can be modeled as a tapped-delay line with $L_{ch} = 3$ delay taps. The channel gains of the taps are $[0 \ -3 \ -8]$ dB and the bandwidth of the HF channel is 24KHz. For both of the proposed models the number of subcarriers per frame is $N=256$ and pilot tone spacing is $N_P = 8$.
the CP-based channel synchronization. Both OFDM and SC-FDE systems experience the impacts of violent frequency selective fading channels even so there are certain contrasts between the performance of their decoders. For lower code rates such as $R = 1/3$ Turbo code; OFDM out-performs SC-FDE. For SC-FDE, the noise amendment loss increases with the average input SNR. When the channel is ineffective and the SNR is high, the equalizer tries harder to invert the nulls and, as a result, the noise in this null locations is amplified. Conversely, OFDM combines the useful energy across all subcarriers through coding and interleaving (Turbo Code).

In the second simulation the effect of the channel delay spread, that is modeled with zero padding in each propagation path is analyzed. It is assumed that no CFO and phase noise exist. The simulation results are shown in the range of 3ms to 10ms channel delay spread. The $R = 1/3$ rate, 4 state (7,5) convolutional turbo encoder has $d_{free}$. Therefore, a coded OFDM system with this turbo code can achieve a diversity order of 5 without implementing any additional antennas, or using any other diversity techniques. Hence lower rate codes are required to achieve full diversity in OFDM systems, especially when the channel order is larger. When this is the case, OFDM gives better performance than SC-FDE system because of the less effect of ISI.

For the third simulation, the effect of random fluctuations in the phase of a wave form due to the VCO at the -140dB/Hz, -100dB/Hz, and -70dB/Hz values is analyzed. For all simulations it can be seen that increasing the CFO, STO and phase noise effect clearly decreases the system performances especially for SC-FDE.

VI. CONCLUSION

In this paper, the performances of the SC and OFDM systems using FDE, MMSE channel estimation, CP-based synchronization over the Wideband HF channel are simulated. The performance of the proposed systems were compared under only CFO, STO and phase noise effects. The simulation results show that the performance of the OFDM system over the HF channel is better than the performance of the SC system. When there is a frequency selective in the channel OFDM can be used to combat ISI and therefore can simplify the code design problem for frequency selective channel. Using a CP, OFDM converts a frequency selective channel into parallel flat fading channels. In this paper we showed the turbo code and OFDM can be combined to achieve a high diversity order. We note that for coded-OFDM performs significantly better than SC-FDE, especially in channels with large diversity.

REFERENCES