A Maximum-Likelihood Bound Approaching Minimum Bit Error Rate Linear Multiuser Detector for Frequency-Selective DS-CDMA Systems

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Abstract

In this letter, a minimum bit error rate (MBER) linear multiuser detector (MUD) is considered for Direct-Sequence Code-Division Multiple-Access (DS-CDMA) communication systems, distorted by time-varying and frequency-selective multipath fading channels. Based on the approach for finding filter coefficients of the proposed MBER MUD, an efficient Newton method with a barrier parameter is developed. The BER performance of the MBER MUD is compared to other conventional detectors. The study finds that the proposed MBER MUD has more than 2 dB gain over the linear minimum mean-squared error (LMMSE) detector. Furthermore, in the high SNR region, the BER performance of the proposed MBER MUD approaches the performance of the maximum-likelihood (ML) detector.

Keywords: Constrained optimization problem, linear multiuser detector, minimum bit error rate, Newton’s method

1. Introduction

Multiuser detection [1] deals with demodulating users’ information signal modulating onto their unique signature waveforms from multiuser interference. In DS-CDMA communications system, the performance is mainly limited by multiple access interference (MAI) and Near-Far problem. Verdu, proposed the optimal detector (maximum likelihood sequence (MLSE) detector) that minimizes the probability of error due to MAI [2]. However, the practical implementation of the LMMSE detector is limited by the decoding complexity which is exponential in the number of active users. Therefore, suboptimal linear MUDs such as decorrelating (DEC) and LMMSE with less complexity have been proposed. These suboptimal linear MUDs suppress the MAI [3] and minimize the mean-squared error (MSE) [4] respectively besides achieve optimal near-far resistance. However, these detectors are not capable of providing the lowest bit error rate (BER) among linear detectors. The MBER MUD that alter the filter coefficients of the linear detectors in order to minimize the BER cost function can offer better performance than the DEC and the LMMSE detectors in such scenarios.

Existing considerations such as [5–9] are based on the AWGN channel. Nevertheless, some of these proposed algorithms may not guarantee whether the solutions find the global minimum. Differently from these studies, the concentration is given in this work to the MBER multiuser detection problem for time-varying and Rayleigh frequency-selective fading channels [10]. It should be noted that the BER cost function of the system is highly non-linear and several local minimums may exist. The BER cost function can be converted into a constrained optimization problem with appropriate constraints that always include the global minimum. In order to make the iterative process quicker to be handled for finding the filter coefficient of the MBER MUD, an efficient Newton method including an adaptive barrier parameter is developed. Computer simulation result demonstrates the effectiveness of the proposed algorithm.

2. Preliminaries

We consider here a reverse link transmission DS-CDMA system model, shown in Fig. 1, with K active users and N chips per bit for binary signalling (binary phase shift keying (BPSK)). The total transmitted signal can be expressed as follows:

$$s(t) = \sum_{k=1}^{K} \sqrt{2P_k} a_k(t) b_k(t) \cos(w_c t)$$  \hspace{1cm} (1)$$

where $P_k$ is the signal power, $a_k(t) \in \{\pm 1\}$ is the unique spreading code, $b_k(t) \in \{\pm 1\}$ is the data sequence of the $k^{th}$ user, and $w_c$ is the carrier frequency of the system,
n(t) is the AWGN signal with zero mean and variance σ².

The complex lowpass impulse response of the time-varying and Rayleigh frequency-selective fading channel with L resolvable paths is expressed as:

\[ h_k(t, τ) = \sum_{l=1}^{L} \alpha_{k,l} e^{-jθ_{k,l}} g(t, τ_{k,l}) \otimes δ(t - τ_{k,l}) \]  

(2)

where \( δ(\cdot) \) denotes the Dirac Delta function, \( L \) is the number of multipath between the base station and the desired user, \( ℓ \) is the path index, and \( \{α_{k,l}\}_{ℓ=1}^{L}, \{θ_{k,l}\}_{ℓ=1}^{L} \) are the random path amplitude, random path phase, and random path delay respectively. \( g(t, τ) \) is the impulse response of the time-varying frequency selective fading channel, and the symbol \( \otimes \) denotes convolution in time domain.

The discrete-time received signal vector after chip-rate sampling can be expressed in matrix form as,

\[ r = SHAb + n \]  

(3)

where \( r = [r_1, r_2, ..., r_N]^T \) is the received signal, \( S = [S_1 ... S_K] \) is a \( NxK \) matrix whose columns include the users’ spreading codes, \( H = \text{diag}(h_1, h_2, ..., h_K) \) is the channel matrix, \( h_k = [α_{k,1}, α_{k,2}, ..., α_{k,L}]^T \) is the channel coefficient vector, \( A = \text{diag}(\sqrt{2P_1}, \sqrt{2P_2}, ..., \sqrt{2P_K}) \) is the diagonal amplitude matrix, \( b = [b_1, b_2, ..., b_K]^T \) is the users’ data vector, and \( n \) is the N-dimensional Gaussian noise vector with zero mean and variance \( σ^2 \) and it is independent of the signal sources. The amplitude matrix \( A \) is time-invariant and the Doppler affected \( θ_{k,l} \) of \( h_k \) is time-variant. The linear multiuser detector can be regarded as an FIR filter and its output is given below;

\[ y = w^Tr \]  

(4)

where \( w = [w_1, w_2, ..., w_N]^T \) denotes the coefficient vector for the proposed linear MBER MUD of length \( N \).

3. The Linear MBER MUD

The BER cost function for the user 1 can be written as [9, 11];

\[ P(w) = \frac{1}{2^{K-1}} \sum_{k=1}^{K} Q \left( \frac{A_kS_kH_kw^Tb_k}{σ \| w \|} \right); 1 ≤ k ≤ 2^{K-1} \]  

(5)

where \( Q(x) = \frac{1}{\sqrt{2π}} \int_{-∞}^{x} e^{-u^2/2} du \) is the Gaussian cumulative distribution function. Any \( w \) which minimizes the BER cost function \( P(w) \) in Eq. (5) can be referred as the MBER MUD and the purpose of this study is finding the \( w^* \). Owing to the complex, irregular shape of the BER cost function, as shown in Fig. 2, there is no closed-form expression for \( w^* \) well, then the next equation can be used to convert Eq. (5) into a constrained optimization problem [11, 12];

\[ w_{\text{MBER}} = \text{argmin} \ P(w) \]  

(6)

\[ \min \ P(w) = \frac{1}{2^{K-1}} \sum_{k=1}^{K} Q \left( \frac{A_kS_kH_kw^Tb_k}{σ \| w \|} \right) \]  

(7)

s.t. \( \frac{A_kS_kH_kw^Tb_k}{σ \| w \|} ≥ 0; \quad 1 ≤ k ≤ 2^{K-1} \)

Since the BER cost function is independent of the coefficient vector’s length, the BER cost function can be analyzed on the set \( \| w \| = 1 \)

\[ \min \ P(w) = \frac{1}{2^{K-1}} \sum_{k=1}^{K} Q \left( \frac{w^Tr_k}{σ} \right) \]  

(8)

s.t. \( \| w \| = 1, \frac{w^Tr_k}{σ} ≥ 0; \quad 1 ≤ k ≤ 2^{K-1} \)
Any local minimum of the BER cost function in Eq. (8) is the global minimum. Furthermore, with the constraint \( \| w \| = 1 \), the global minimum is unique [9]. Because of the feasible region which is characterized by constraints in Eq. (8) is not convex, the BER cost function in Eq. (8) do not match for convex programming problem. However, it can be easily converted to convex programming problem with the constraint \( \| w \| \leq 1 \). Since the solution of the constrained optimization problem in Eq. (8) corresponds to the feasible region which is characterized by constraints in Eq. (9).

In Eq. (9), the non-linear constraint can be dropped and converted the equation into the form as given below:

\[
\min \quad P(w) = \frac{1}{2K-1} \sum_{k=1}^{K} Q \left( \frac{w^T r_k}{\sigma} \right) 
\]

\[
s.t. \quad \| w \| \leq 1, \quad \frac{w^T r_k}{\sigma} \geq 0; \quad 1 \leq k \leq 2^{K-1}
\]

The difference between Eq. (8) and Eq. (9) is that the latter one is the convex programming problem where several algorithms are available for. The proposed minimization algorithm includes three sections such as: constructing a barrier parameter by using adaptive barrier method, finding the convergence direction with Golden-Section method, and choosing suitable initial vector within the feasible region.

\[
\min \quad F_\mu(w) = P(w) - \mu \log (1 - w^T w) 
\]

\[
s.t. \quad \frac{w^T r_k}{\sigma} \geq 0; \quad 1 \leq k \leq 2^{K-1}
\]

where \( \mu \) is a barrier parameter and \( \mu > 0 \). If the initial coefficient vector is chosen in the feasible region, it satisfies the constraints in Eq. (9). The Gradient and the Hessian of the \( F_\mu(w) \) are given by in [9]:

\[
\nabla F_\mu(w) = -\sum_{i=1}^{K-1} \frac{1}{2^{K-1}} e^{-\lambda_i/2} r_i + \frac{2\mu w}{1-\|w\|^2} \tag{11}
\]

\[
\nabla^2 F_\mu(w) = \sum_{i=1}^{K-1} \frac{1}{2^{K-1}} e^{-\lambda_i/2} r_i r_i^T + \frac{2\mu}{1-\|w\|^2} \tag{12}
\]

where \( \lambda_i = w^T r_i \) and \( I \) is the unit matrix. Given the gradient and the hessian of \( F_\mu(w) \) with respect to \( w \), \( \nabla F_\mu(w) \), and \( \nabla^2 F_\mu(w) \), a MBER iteration procedure can be explained as:

\[
w_{k+1} = w_k + \zeta_k \Omega_k \tag{13}
\]

where \( \Omega_k \) is the search direction and it can be obtained using:

\[
\Omega_k = - \left[ \nabla^2 F_\mu(w) \right]^{-1} \nabla F_\mu(w) \tag{14}
\]

The positive scalar \( \zeta \) can be determined as follows [10].

\[
\zeta = \left[ (w_k^T r_i)^2 - \| \Omega_k \|^2 (\| \Omega_k \|^2 - 1) \right]^{1/2} - w_k^T \Omega_k \tag{15}
\]

The iterative procedure for proposed method is given below:

**Algorithm 1** The Linear MBER MUD Algorithm

- **Initialization**
  - Determine \( s, H, A, b \).
  - Choose \( \mu \) and \( w_0 \).
  - Determine an error parameter \( \varepsilon \) which stops the iterations.

- **Loop**
  - Set \( w_k = w_0 \).
  - Compute the \( \Omega_k \) at \( w_k \) by using (11), (12), and (14).
  - Determine \( \zeta_k \) by using Goldens-section method (15).
  - Calculate the next value of \( w_k \) by using \( w_{k+1} = w_k + \zeta_k \Omega_k \).

- **Stop**
  - If \( P(w_{k+1}) < P(w) \) and \( \| w_{k+1} - w_k \| < \varepsilon \), then \( w_{k+1} \) is the optimal solution. Stop and set \( w_{k+1} = w^* \). Otherwise set \( k = k + 1 \) and go to Loop.

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### 4. Numerical Example

A five-user synchronous system with \( N=31 \) chips Gold-Codes for the users’ signature sequences is considered and a time-varying Rayleigh frequency-selective fading channel model is employed. The main purpose of the example is to illustrate how the MBER MUD outperforms the DEC, and the LMMSE detectors. The BER performance of the proposed MUD is also compared with the optimal (ML) detectors. Random channel parameters are used for varying
the channel in the time domain. Assumptions are perfect knowledge of the channel at the detectors for uplink and perfect power control. In the Monte-Carlo simulations, the MBER MUD was taken as the global minimum for Eq. (9) of 45 runs of the proposed minimization algorithm. Fig. 3 shows the average BER curves of several detectors for user 1.

The study finds that the proposed the MBER MUD outperforms the LMMSE and the DEC multiuser detectors by about 2 dB and 3.6 dB respectively. Observe that the LMMSE and the DEC curves are nearly indistinguishable for low values of SNR. On the other hand, the MBER MUD’s curve is drastically different from the other curves for especially high SNR region. Note that, in the high SNR region, the performance of the MBER MUD was nearly equal to that of the maximum-likelihood (ML) detector [1]. The BER values of the multiuser detectors for the values of SNR are given in Table 1.

5. Conclusions

The MBER MUD for time-varying and Rayleigh frequency-selective channels on DS-CDMA system is considered. The BER cost function of multiuser detection for time-varying Rayleigh frequency-selective fading DS-CDMA channels is studied and converted into an equivalent constrained optimization problem. The proposed algorithm minimizes the BER cost function directly subject to the constraint that is a convex set of the feasible region. The Newton’s method with a barrier parameter is used to find the filter coefficients of the proposed MBER MUD, which has lower computational complexity than the ML detector [2].

The analysis and simulations results confirm that the proposed MBER MUD minimizes the BER cost function and can achieve lower BER than the decorrelating and the LMMSE detectors in medium or high SNR cases. Furthermore, the MBER MUD has a lower complexity than the ML multiuser detector but achieves approximately similar BER performance.

References